Chapter 5

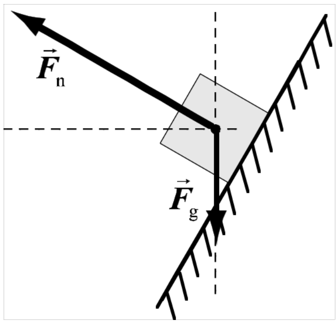
Problems: 1, 8, 17, 31, 34, 40, 50, 67, 68, 81, 83, 102, 121 and 129
Think about: 6, 16, 87, 110 and 136

1 • **[SSM]** Various objects lie on the bed of a truck that is moving along a straight horizontal road. If the truck gradually speeds up, what force acts on the objects to cause them to speed up too? Explain why some of the objects might stay stationary on the floor while others might slip backward on the floor.

Determine the Concept The forces acting on the objects are the normal and frictional forces exerted by the truck bed and the gravitational force exerted by Earth. The static (if the objects do not slip) frictional forces exerted by the floor of the truck bed cause them to speed up. Because the objects are speeding up (accelerating), there must be a net force acting on them. Of these forces, the only one that acts in the direction of the acceleration is the static friction force. The maximum acceleration is determined not by the mass of the objects but instead by the value of the coefficient of static friction. This will vary from object to object depending on its material and surface characteristics.

6 •• If it is started properly on the frictionless inside surface of a cone (Figure 5-57), a block is capable of maintaining uniform circular motion. Draw the free-body diagram of the block and identify clearly which force (or forces or force components) is responsible for the centripetal acceleration of the block.

Determine the Concept The forces acting on the block are the normal force $\vec{F}_\text{n}$ exerted by the surface of the cone and the gravitational force $\vec{F}_\text{g}$ exerted by the earth. The horizontal component of $\vec{F}_\text{n}$ is responsible for the centripetal acceleration on the block.



8 • Viewed from an inertial reference frame, an object is seen to be moving in a circle. Which, if any, of the following statements are true.
(*a*) A non-zero net force is acting acts the object. (*b*) The object cannot have a radially outward force acting on it. (*c*) At least one of the forces acting on the object must point directly toward the center of the circle.

(*a*) True. The velocity of an object moving in a circle is continually changing independently of whether the object's speed is changing. The change in the velocity vector and the acceleration vector and the net force acting on the object all point toward the center of circle. This center-pointing force is called a centripetal force.
(*b*) False. The only condition that must be satisfied in order that the object move along a circular path is that the *net* force acting on it be radially inward.
(*c*) False. The only condition that must be satisfied in order that the object move

along a circular path is that the *net* force acting on it be radially inward.

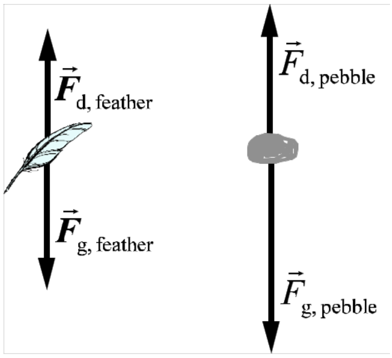
16 • A block is sliding on a frictionless surface along a loop-the-loop, as in Figure 5-59. The block is moving fast enough so that it never loses contact with the track. Match the points along the track to the appropriate free-body diagrams in the figure.

Determine the Concept The only forces acting on the block are its weight and the force the surface exerts on it. Because the loop-the-loop surface is frictionless, the force it exerts on the block must be perpendicular to its surface.

- ✓ At point A the weight is downward and the normal force is to the right. The normal force is the centripetal force. Free-body diagram 3 matches these forces.
- ✓ At point B the weight is downward, the normal force is upward, and the normal force is greater than the weight so that their difference is the centripetal force. Free-body diagram 4 matches these forces.
- ✓ At point C the weight is downward and the normal force is to the left. The normal force is the centripetal force. Free-body diagram 5 matches these forces.
- ✓ At point D both the weight and the normal forces are downward. Their sum is the centripetal force. Free-body diagram 2 matches these forces.

17 •• **[SSM]** (*a*) A pebble and a feather held at the same height above the ground are simultaneously dropped. During the first few milliseconds following release the drag force on the pebble is less than that on the feather, but later on during the fall the *opposite* is true. Explain. (*b*) In light of this result, explain how the pebble's acceleration can be so obviously larger than that of the feather. (*Hint*: *Draw a free-body diagram of each object*.)

Determine the Concept The drag force acting on the objects is given by $F_d = \frac{1}{2}CA\rho v^2$, where A is the projected surface area, v is the object's speed, ρ is the density of air, and C is a dimensionless coefficient. We'll assume that, over the height of the fall, the density of air ρ is constant. The free-body diagrams for a feather and a pebble several milliseconds into their fall are shown to the right. The forces acting on both objects are the downward gravitational force $\vec{F}_g$ and an upward drag force $\vec{F}_d$.



(*a*) The drag force on an object is proportional to some power of its speed. For a millisecond or two following release, the speeds of both the pebble and the feather are negligible, so the drag forces are negligible and they both fall with the same free-fall acceleration *g*. During this brief period their speeds remain equal, so the object that presents the greater area has the greater drag force. It is the feather that presents the greater area, so during this brief period the drag force on the feather is greater than that on the pebble.

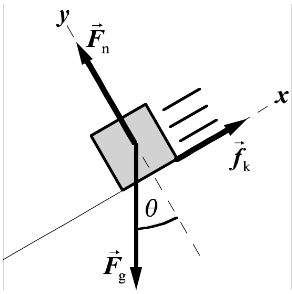
A short time after the initial period the feather reaches terminal speed, after which the drag force on it remains equal to the gravitational force on it. However, the gravitational force on the pebble is much greater than that on the feather, so the pebble continues to gain speed long after the feather reaches terminal speed. As the pebble continues to gain speed, the drag force on it continues to increase. As a result, the drag force on the pebble eventually exceeds the drag force on the feather.

(*b*) The acceleration of the feather rapidly decreases because the drag force on it approaches the gravitational force on it shortly after release. However, the drag force on the pebble does not approach the gravitational force on it until much higher speeds are attained, which means the acceleration of the pebble remains high for a longer period of time.

31 • **[SSM]** A block of mass *m* slides at constant speed down a plane inclined at an angle of θ with the horizontal. It follows that (*a*) $\mu_k = mg \sin \theta$, (*b*) $\mu_k = \tan \theta$, (*c*) $\mu_k = 1 - \cos \theta$, (*d*) $\mu_k = \cos \theta - \sin \theta$.

Picture the Problem The block is in equilibrium under the influence of $\vec{F}_n$, $m\vec{g}$, and $\vec{f}_k$; that is, $\vec{F}_n + m\vec{g} + \vec{f}_k = 0$. We can apply Newton's 2nd law to determine the relationship between f_k, θ, and mg.

A pictorial representation showing the forces acting on the sliding block is shown to the right.



Using its definition, express the coefficient of kinetic friction:

$$\mu_k = \frac{f_k}{F_n} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block:

$$f_k - mg \sin\theta = ma_x$$

or, because $a_x = 0$,

$$f_k = mg \sin\theta$$

Apply $\sum F_y = ma_y$ to the block:

$$F_\text{n} - mg\cos\theta = ma_y$$

or, because $a_y = 0$,

$$F_\text{n} = mg\cos\theta$$

Substitute for f_k and F_n in equation (1) and simplify to obtain:

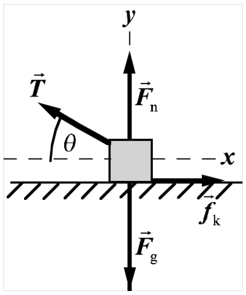
$$\mu_\text{k} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

and $\boxed{(b)}$ is correct.

34 • A block of mass *m* is pulled at a constant velocity across a horizontal surface by a string as shown in Figure 5-60. The magnitude of the frictional force is (*a*) $\mu_\text{k}\, mg$, (*b*) $T\cos\theta$, (*c*) $\mu_\text{k}\,(T - mg)$, (*d*) $\mu_\text{k}\, T\sin\theta$, or (*e*) $\mu_\text{k}\,(mg - T\sin\theta)$.

Picture the Problem The block is in equilibrium under the influence of the forces $\vec{T}$, $\vec{f}_\text{k}$, $\vec{F}_n$, and $\vec{F}_g$; that is $\vec{T} + \vec{f}_\text{k} + \vec{F}_g + \vec{F}_n = 0$. We can apply Newton's 2nd law to determine the relationship between T and f_k.

A free-body diagram showing the forces acting on the block is shown to the right.



Apply $\sum F_x = ma_x$ to the block:

$$-T\cos\theta + f_\text{k} = ma_x$$

Because $a_x = 0$:

$f_\text{k} = T\cos\theta$ and $\boxed{(b)}$ is correct.

Apply $\sum F_y = ma_y$ to the block:

$$F_n + T\sin\theta - mg = ma_y = 0$$

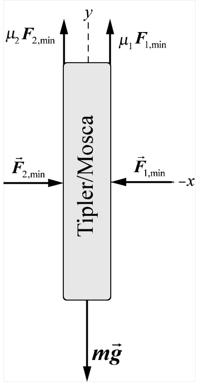
Because $a_y = 0$:

$f_\text{k} = \mu_k F_n = \mu_k(mg - T\sin\theta)$ and $\boxed{(e)}$ is correct.

40 •• A tired and overloaded student is attempting to hold a large physics textbook wedged under his arm, as shown in Figure 5-61. The textbook has a mass of 3.2 kg, while the coefficient of static friction of the textbook against the student's underarm is 0.320 and the coefficient of static friction of the book against the student's shirt is 0.160. (*a*) What is the minimum horizontal force that the student must apply to the textbook to prevent it from falling? (*b*) If the student can only exert a force of 61 N, what is the acceleration of the textbook as it slides from under his arm? The coefficient of kinetic friction of arm against textbook is 0.200, while that of shirt against textbook is 0.090.

Picture the Problem We can apply Newton's 2^{nd} law to relate the minimum force required to hold the book in place to its mass and to the coefficients of static friction. In Part (*b*), we can proceed similarly to relate the acceleration of the book to the coefficients of kinetic friction.

(*a*) The force diagram shows the forces acting on the book. The normal force is the net force the student exerts in squeezing the book. Let the horizontal direction be the *x* direction and upward the *y* direction. Note that the normal force is the same on either side of the book because it is not accelerating in the horizontal direction. The book could be accelerating downward.



Apply $\sum \vec{F} = m\vec{a}$ to the book:

$$\sum F_x = F_{2,\min} - F_{1,\min} = 0$$

and

$$\sum F_y = \mu_{s,1} F_{1,\min} + \mu_{s,2} F_{2,\min} - mg = 0$$

Noting that $F_{1,\min} = F_{2,\min}$, solve the y equation for $F_{\min}$:

$$F_{\min} = \frac{mg}{\mu_{s,1} + \mu_{s,2}}$$

Substitute numerical values and evaluate $F_{\min}$:

$$F_{\min} = \frac{(3.2\,\text{kg})(9.81\,\text{m/s}^2)}{0.320 + 0.160} = \boxed{65\,\text{N}}$$

(*b*) Apply $\sum F_y = ma_y$ with the book accelerating downward, to obtain:

$$\sum F_y = \mu_{k,1} F + \mu_{k,2} F - mg = ma$$

Solving for a_y yields:

$$a_y = \frac{\mu_{k,1} + \mu_{k,2}}{m} F - g$$

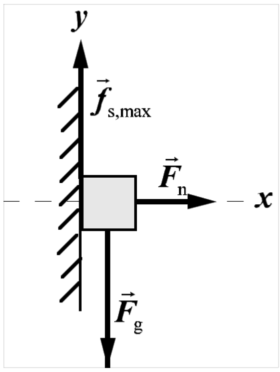
Substitute numerical values and evaluate a_y:

$$a_y = \left| \left(\frac{0.200 + 0.090}{3.2\,\text{kg}} \right) (61\,\text{N}) - 9.81\,\text{m/s}^2 \right|$$

$$= \boxed{4.3\,\text{m/s}^2\text{, downward}}$$

50 •• You and your best pal make a friendly bet that you can place a 2.0-kg box against the side of a cart, as in Figure 5-63, and that the box will not fall to the ground, even though you guarantee to use no hooks, ropes, fasteners, magnets, glue, or adhesives of any kind. When your friend accepts the bet, you begin pushing the cart in the direction shown in the figure. The coefficient of static friction between the box and the cart is 0.60. (*a*) Find the minimum acceleration for which you will win the bet. (*b*) What is the magnitude of the frictional force in this case? (*c*) Find the force of friction on the box if the acceleration is twice the minimum needed for the box not to fall. (*d*) Show that, for a box of any mass, the box will not fall if the magnitude of the forward acceleration is $a \geq g/\mu_s$, where μ_s is the coefficient of static friction.

Picture the Problem To hold the box in place, the acceleration of the cart and box must be great enough so that the static friction force acting on the box will equal the weight of the box. We can use Newton's 2nd law to determine the minimum acceleration required.



(*a*) Noting that $F_g = mg$, apply $\sum \vec{F} = m\vec{a}$ to the box:

$$\sum F_x = F_n = ma_{\min,x} \qquad (1)$$

and

$$\sum F_y = f_{s,\max} - mg = 0 \qquad (2)$$

Substituting $\mu_s F_n$ for $f_{s,\max}$ in equation (2) yields:

$$\mu_s F_n - mg = 0$$

Substitute for F_n from equation (1) to obtain:

$$\mu_s(ma_{\min,x}) - mg = 0 \Rightarrow a_{\min,x} = \frac{g}{\mu_s}$$

Substitute numerical values and evaluate $a_{\min,x}$:

$$a_{\min,x} = \frac{9.81\,\text{m/s}^2}{0.60} = \boxed{16\,\text{m/s}^2}$$

(*b*) From equation (2) we have:

$$f_{s,\max} = mg$$

Substitute numerical values and evaluate $f_{s,\max}$:

$$f_{s,\max} = (2.0\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{20\,\text{N}}$$

(*c*) If a is twice that required to hold the box in place, f_s will still have its maximum value given by:

$$f_{s,\max} = \boxed{20\,\text{N}}$$

(*d*) Because $a_{\min,x} = g/\mu_s$, the box will not fall if $a \geq g/\mu_s$.

67 • **[SSM]** A Ping-Pong ball has a mass of 2.3 g and a terminal speed of 9.0 m/s. The drag force is of the form bv^2. What is the value of b?

Picture the Problem The ping-pong ball experiences a downward gravitational force exerted by the earth and an upward drag force exerted by the air. We can apply Newton's 2nd law to the Ping-Pong ball to obtain its equation of motion. Applying terminal speed conditions will yield an expression for b that we can evaluate using the given numerical values. Let the downward direction be the $+y$ direction.

Apply $\sum F_y = ma_y$ to the Ping-Pong ball:

$$mg - bv^2 = ma_y$$

When the Ping-Pong ball reaches its terminal speed $v = v_t$ and $a_y = 0$:

$$mg - bv_t^2 = 0 \Rightarrow b = \frac{mg}{v_t^2}$$

Substitute numerical values and evaluate b:

$$\boldsymbol{b} = \frac{(2.3\times10^{-3}\text{ kg})(9.81\text{ m/s}^2)}{(9.0\text{ m/s})^2} = \boxed{2.8\times10^{-4}\text{ kg/m}}$$

68 • A small pollution particle settles toward Earth in still air. The terminal speed is 0.30 mm/s, the mass of the particle is 1.0×10^{-10} g and the drag force is of the form bv. What is the value of b?

Picture the Problem The pollution particle experiences a downward gravitational force exerted by the earth and an upward drag force exerted by the air. We can apply Newton's 2nd law to the particle to obtain its equation of motion. Applying terminal speed conditions will yield an expression for b that we can evaluate using the given numerical values. Let the downward direction by the $+y$ direction.

Apply $\sum F_y = ma_y$ to the particle:

$$mg - bv = ma_y$$

When the particle reaches its terminal speed $v = v_t$ and $a_y = 0$:

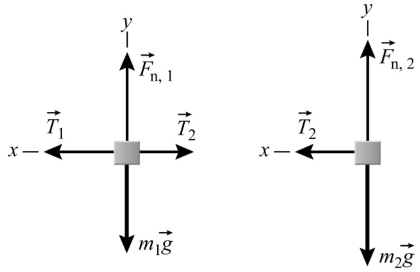
$$mg - bv_t = 0 \Rightarrow b = \frac{mg}{v_t}$$

Substitute numerical values and evaluate b:

$$b = \frac{(1.0\times10^{-13}\text{ kg})(9.81\text{ m/s}^2)}{3.0\times10^{-4}\text{ m/s}} = \boxed{3.3\times10^{-9}\text{ kg/s}}$$

81 •• **[SSM]** A block of mass m_1 is attached to a cord of length L_1, which is fixed at one end. The block moves in a horizontal circle on a frictionless tabletop. A second block of mass m_2 is attached to the first by a cord of length L_2 and also moves in a circle on the same frictionless tabletop, as shown in Figure 5-73. If the period of the motion is T, find the tension in each cord in terms of the given symbols.

Picture the Problem The free-body diagrams show the forces acting on each block. We can use Newton's 2nd law to relate these forces to each other and to the masses and accelerations of the blocks.



Apply $\sum F_x = ma_x$ to the block whose mass is m_1:

$$T_1 - T_2 = m_1 \frac{v_1^2}{L_1}$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$T_2 = m_2 \frac{v_2^2}{L_1 + L_2}$$

Relate the speeds of each block to their common period T and their distance from the center of the circle:

$$v_1 = \frac{2\pi L_1}{T} \text{ and } v_2 = \frac{2\pi(L_1 + L_2)}{T}$$

In the second force equation, substitute for v_2, and simplify to obtain:

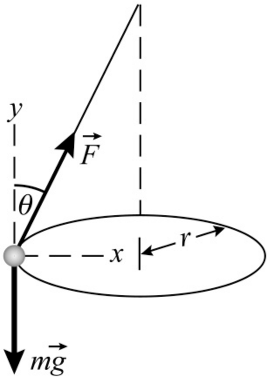
$$T_2 = \boxed{[m_2(L_1 + L_2)]\left(\frac{2\pi}{T}\right)^2}$$

Substitute for T_2 and v_1 in the first force equation to obtain:

$$T_1 = \boxed{[m_2(L_1 + L_2) + m_1 L_1]\left(\frac{2\pi}{T}\right)^2}$$

83 •• You are swinging your younger sister in a circle of radius 0.75 m, as shown in Figure 5-74. If her mass is 25 kg and you arrange it so she makes one revolution every 1.5 s, (*a*) what is the magnitude and direction of the force that must be exerted by you on her? (Assume her to be a point particle.) (*b*) What is the magnitude and direction of the force she exerts on you?

Picture the Problem The diagram to the right has the free-body diagram for the child superimposed on a pictorial representation of her motion. The force you exert on your sister is $\vec{F}$ and the angle it makes with respect to the direction we've chosen as the positive y direction is θ. We can infer her speed from the given information concerning the radius of her path and the period of her motion. Applying Newton's 2nd law will allow us to find both the direction and magnitude of $\vec{F}$.



(a) Apply $\sum \vec{F} = m\vec{a}$ to the child:

$$\sum F_x = F\sin\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F\cos\theta - mg = 0$$

Eliminate F between these equations and solve for θ to obtain:

$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

Express v in terms of the radius and period of the child's motion:

$$v = \frac{2\pi r}{T}$$

Substitute for v in the expression for θ to obtain:

$$\theta = \tan^{-1}\left[\frac{4\pi^2 r}{gT^2}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[\frac{4\pi^2(0.75\,\text{m})}{(9.81\,\text{m/s}^2)(1.5\,\text{s})^2}\right] = 53.3°$$

$$= \boxed{53°}\ \text{above horizontal}$$

Solve the y equation for F:

$$F = \frac{mg}{\cos\theta}$$

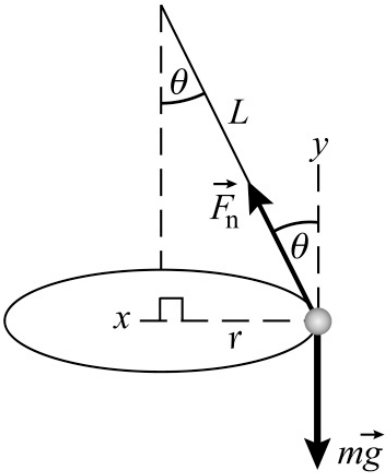
Substitute numerical values and evaluate F:

$$\boldsymbol{F} = \frac{(25\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 53.3°} = \boxed{0.41\,\text{kN}}$$

(b) The force your sister exerts on you is the reaction force to the force you exert on her. Thus its magnitude is the same as the force you exert on her (0.41 kN) and its direction is 53° below horizontal.

87 ••• A small bead with a mass of 100 g (Figure 5-75) slides without friction along a semicircular wire with a radius of 10 cm that rotates about a vertical axis at a rate of 2.0 revolutions per second. Find the value of θ for which the bead will remain stationary relative to the rotating wire.

Picture the Problem The semicircular wire of radius 10 cm limits the motion of the bead in the same manner as would a 10-cm string attached to the bead and fixed at the center of the semicircle. The horizontal component of the normal force the wire exerts on the bead is the centripetal force. The application of Newton's 2nd law, the definition of the speed of the bead in its orbit, and the relationship of the frequency of a circular motion to its period will yield the angle at which the bead will remain stationary relative to the rotating wire.



Apply $\sum \vec{F} = m\vec{a}$ to the bead:

$$\sum F_x = F_n \sin\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - mg = 0$$

Eliminate F_n from the force equations to obtain:

$$\tan\theta = \frac{v^2}{rg}$$

The frequency of the motion is the reciprocal of its period T. Express the speed of the bead as a function of the radius of its path and its period:

$$v = \frac{2\pi r}{T}$$

Using the diagram, relate r to L and θ:

$$r = L\sin\theta$$

Substitute for r and v in the expression for $\tan\theta$ and solve for θ:

$$\theta = \cos^{-1}\left[\frac{gT^2}{4\pi^2 L}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left[\frac{(9.81\,\text{m/s}^2)(0.50\,\text{s})^2}{4\pi^2(0.10\,\text{m})}\right] = \boxed{52°}$$

102 • On a weekend archeological dig, you discover an old club-ax that consists of a symmetrical 8.0-kg stone attached to the end of a uniform 2.5-kg stick. You measure the dimensions of the club-ax as shown in Figure 5-77. How far is the center of mass of the club-ax from the handle end of the club-ax?

Picture the Problem Let the left end of the handle be the origin of our coordinate system. We can disassemble the club-ax, find the center of mass of each piece, and then use these coordinates and the masses of the handle and stone to find the center of mass of the club-ax.

Express the center of mass of the handle plus stone system:

$$x_{\text{cm}} = \frac{m_{\text{stick}}x_{\text{cm,stick}} + m_{\text{stone}}x_{\text{cm,stone}}}{m_{\text{stick}} + m_{\text{stone}}}$$

Assume that the stone is drilled and the stick passes through it. Use symmetry considerations to locate the center of mass of the stick:

$$x_{\text{cm,stick}} = 49\,\text{cm}$$

Use symmetry considerations to locate the center of mass of the stone:

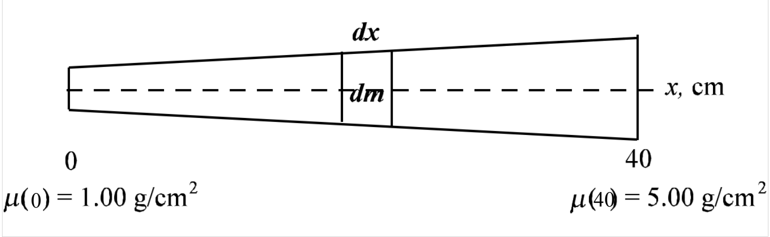
$$x_{\text{cm,stone}} = 89\,\text{cm}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{\text{cm}} = \frac{(2.5\,\text{kg})(49\,\text{cm}) + (8.0\,\text{kg})(89\,\text{cm})}{2.5\,\text{kg} + 8.0\,\text{kg}} = \boxed{79\,\text{cm}}$$

110 •• Find the location of the center of mass of a nonuniform rod 0.40 m in length if its density varies linearly from 1.00 g/cm at one end to 5.00 g/cm at the other end. Specify the center-of-mass location relative to the less-massive end of the rod.

Picture the Problem The pictorial representation summarizes the information we're given about the non-uniform rod. We can use the definition of the center of mass for a continuous object to find the center of mass of the non-uniform rod.



The x coordinate of the center of mass of the non-uniform rod is given by:

$$x_{\text{cm}} = \frac{\int x\,dm}{\int dm}$$

or, because $\boldsymbol{dm} = \boldsymbol{\mu}(\boldsymbol{x})\boldsymbol{dx}$,

$$\boldsymbol{x}_{\text{cm}} = \frac{\int \boldsymbol{x\mu}(\boldsymbol{x})\boldsymbol{dx}}{\int \boldsymbol{\mu}(\boldsymbol{x})\boldsymbol{dx}} \qquad (1)$$

By symmetry:

$$\boldsymbol{y}_{\text{cm}} = 0$$

Use the given information regarding the linear variation in the density of the non-uniform rod to express $\mu(x)$:

$$\boldsymbol{\mu}(\boldsymbol{x}) = 1.00\ \text{g/cm} + \left(0.10\ \text{g/cm}^2\right)\boldsymbol{x}$$

Substituting for $\mu(x)$ in equation (1) yields:

$$x_{\text{cm}} = \frac{\int_0^{40\ \text{cm}} x\left[1.00\ \text{g/cm} + \left(0.10\ \text{g/cm}^2\right)x\right]dx}{\int_0^{40\ \text{cm}} \left[1.00\ \text{g/cm} + \left(0.10\ \text{g/cm}^2\right)x\right]dx}$$

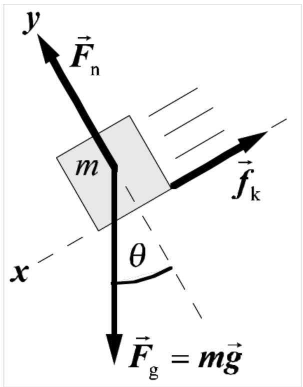
Evaluate these integrals to obtain:

$$x_{\text{cm}} = \frac{2933.3\ \text{g}\cdot\text{cm}}{120\ \text{g}} = 24.4\ \text{cm}$$

The coordinates of the center of mass of the non-uniform rod are $\boxed{(24\ \text{cm}, 0)}$.

121 • A 4.5-kg block slides down an inclined plane that makes an angle of 28° with the horizontal. Starting from rest, the block slides a distance of 2.4 m in 5.2 s. Find the coefficient of kinetic friction between the block and plane.

Picture the Problem The forces that act on the block as it slides down the incline are shown on the free-body diagram to the right. The acceleration of the block can be determined from the distance-and-time information given in the problem. The application of Newton’s 2nd law to the block will lead to an expression for the coefficient of kinetic friction as a function of the block’s acceleration and the angle of the incline.



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\sum F_x = mg\sin\theta - f_\text{k} = ma_x \qquad (1)$$

and

$$\sum F_y = F_\text{n} - mg\cos\theta = 0 \qquad (2)$$

Set $f_k = \mu_k F_n$ in equation (1) to obtain:

$$mg\sin\theta - \mu_k F_n = ma_x \qquad (3)$$

Solve equation (2) for F_n and substitute in equation (3) to obtain:

$$mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

Solving for μ_k yields:

$$\mu_k = \frac{g\sin\theta - a_x}{g\cos\theta} \qquad (4)$$

Using a constant-acceleration equation, relate the distance the block slides to its sliding time:

$$\Delta x = v_{0x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

or, because $v_{0x} = 0$,

$$\Delta x = \tfrac{1}{2}a_x(\Delta t)^2 \Rightarrow a_x = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute for a_x in equation (4) to obtain:

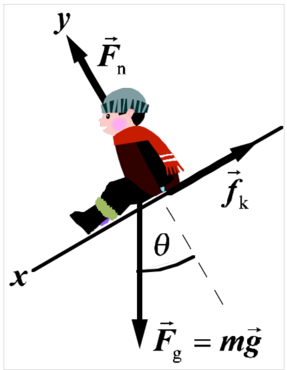
$$\mu_k = \frac{g\sin\theta - \dfrac{2\Delta x}{(\Delta t)^2}}{g\cos\theta}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{(9.81\,\text{m/s}^2)\sin 28° - \dfrac{2(2.4\,\text{m})}{(5.2\,\text{s})^2}}{(9.81\,\text{m/s}^2)\cos 28°} = \boxed{0.51}$$

129 •• A child of mass m slides down a slide inclined at 30° in time t_1. The coefficient of kinetic friction between her and the slide is μ_k. She finds that if she sits on a small sled (also of mass m) with frictionless runners, she slides down the same slide in time $\tfrac{1}{2}t_1$. Find μ_k.

Picture the Problem The following free-body diagram shows the forces acting on the child as she slides down the incline. We'll first use Newton's 2nd law to derive an expression for μ_k in terms of her acceleration and then use Newton's 2nd law to find her acceleration when riding the frictionless cart. Using a constant-acceleration equation, we'll relate these two accelerations to her descent times and solve for her acceleration when sliding. Finally, we can use this acceleration in the expression for μ_k.



Apply $\sum \vec{F} = m\vec{a}$ to the child as she slides down the incline:

$$\sum F_x = mg\sin\theta - f_\mathrm{k} = ma_{1,x}$$

and

$$\sum F_y = F_\mathrm{n} - mg\cos\theta = 0$$

Because $f_\mathrm{k} = \mu_\mathrm{k} F_\mathrm{n}$, the x-equation can be written:

$$mg\sin\theta - \mu_\mathrm{k} F_\mathrm{n} = ma_{1,x} \qquad (1)$$

Solving the y-equation for F_n yields:

$$F_\mathrm{n} = mg\cos\theta$$

Substitute for F_n in equation (1) to obtain:

$$mg\sin\theta - \mu_\mathrm{k} mg\cos\theta = ma_{1,x}$$

Solving for μ_k yields:

$$\mu_\mathrm{k} = \tan 30° - \frac{a_{1,x}}{g\cos 30°} \qquad (2)$$

Apply $\sum F_x = ma_x$ to the child as she rides the frictionless cart down the incline and solve for her acceleration $a_{2,x}$:

$$mg\sin 30° = ma_{2,x}$$

and

$$a_{2,x} = g\sin 30°$$

Letting s represent the distance she slides down the incline, use a constant-acceleration equation to relate her sliding times to her accelerations and distance traveled down the slide :

$$s = v_{0x}t_1 + \tfrac{1}{2}a_{1,x}t_1^2 \text{ where } v_{0x} = 0$$

and

$$s = v_{0x}t_2 + \tfrac{1}{2}a_{2,x}t_2^2 \text{ where } v_{0x} = 0$$

Equate these expressions, substitute $t_2 = \frac{1}{2}t_1$ and solve for $a_{1,x}$:

$$a_{1,x} = \tfrac{1}{4}a_{2,x} = \tfrac{1}{4}g\sin 30°$$

Substitute for $a_{1,x}$ in equation (2) to obtain:

$$\mu_k = \tan 30° - \frac{\frac{1}{4}g\sin 30°}{g\cos 30°} = \tfrac{3}{4}\tan 30°$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \tfrac{3}{4}\tan 30° = \boxed{0.43}$$

136 •• A circular plate of radius R has a circular hole of radius $R/2$ cut out of it (Figure 5-86). Find the center of mass of the plate after the hole has been cut. (*Hint:* The plate can be modeled as two disks superimposed, with the hole modeled as a disk negative mass.)

Picture the Problem By symmetry, $x_{cm} = 0$. Let σ be the mass per unit area of the disk. The mass of the modified disk is the difference between the mass of the whole disk and the mass that has been removed.

Start with the definition of y_{cm}:

$$y_{cm} = \frac{\sum_i m_i y_i}{M - m_{hole}} = \frac{m_{disk}y_{disk} - m_{hole}y_{hole}}{M - m_{hole}}$$

Express the mass of the complete disk:

$$M = \sigma A = \sigma\pi r^2$$

Express the mass of the material removed:

$$m_{hole} = \sigma\pi\left(\frac{r}{2}\right)^2 = \tfrac{1}{4}\sigma\pi r^2 = \tfrac{1}{4}M$$

Substitute and simplify to obtain:

$$y_{cm} = \frac{M(0) - \left(\frac{1}{4}M\right)\left(-\frac{1}{2}r\right)}{M - \frac{1}{4}M} = \boxed{\tfrac{1}{6}r}$$